Hubble Scale from Time-Induced Complexity

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Abstract

The thinning of neighborhoods under added distinguishing structure suggests a natural cosmological program. As a three-dimensional world accumulates a longer history, more relational information is required to specify its present state; the thinning of compatible histories may then acquire a geometric expression. The simplest realization is unexpectedly suggestive: it predicts $H_0 = 1/t_0 = 70.85\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ from the age of the Universe, close to measured values. This note develops the mathematical bridge from the relational-shape theorem to that estimate and then uses cosmological data to identify its next required ingredient. Three spatial history channels imply $a(t) \propto t$, while literal counting of three spatial dimensions plus time implies $a(t) \propto t^{4/3}$. These fixed laws are informative first approximations; DESI DR2 baryon acoustic oscillation distances point beyond them to an evolving rate of independent relational information. A relativistic causal-past construction reinforces the same direction. The result is a concrete scale connection together with a sharply defined route toward a covariant, dynamical model.

The relational-shape theorem and the proposed bridge

The point of departure is the formalism of the companion paper, *The Relational Shape of Structural Complexity and Neighborhood Density*. Let $\mathcal{O} = \{O_1, \ldots, O_K\}$ be a population of $K \geq 2$ objects. At refinement depth N, object O_i is represented by N metadata values,

$$O_i^{(N)} = (o_{i1}, \ldots, o_{iN}),$$

where $i, j \in \{1, \ldots, K\}$ label objects and $n \in \{1, \ldots, N\}$ labels a metadata coordinate. The additive distance is

$$\delta_N(O_i, O_j) = \sum_{n=1}^{N} d_n(o_{in}, o_{jn}), \qquad d_n(x, y) \geq 0, \tag{1}$$

where d_n is the nonnegative dissimilarity contributed by coordinate n. The mean separation of O_i from the rest of the population is

$$D_N(i) = \frac{1}{K-1} \sum_{j \neq i} \delta_N(O_i, O_j). \tag{2}$$

For a tolerance radius $r \geq 0$, define the neighborhood and its normalized density by

$$\mathcal{N}_r^{(N)}(O_i) = \{O_j \in \mathcal{O} : j \neq i,\ \delta_N(O_i, O_j) \leq r\}, \tag{3}$$

$$\rho_N(i; r) = \frac{|\mathcal{N}_r^{(N)}(O_i)|}{K-1} = \frac{1}{K-1} \sum_{j \neq i} \mathbf{1}\{\delta_N(O_i, O_j) \leq r\}, \tag{4}$$

where $|\cdot|$ denotes cardinality and $\mathbf{1}\{\cdot\}$ is an indicator, equal to one when its condition is true and zero otherwise. Because the new coordinate contribution is nonnegative,

$$\delta_{N+1} \geq \delta_N, \qquad D_{N+1}(i) \geq D_N(i), \qquad \mathcal{N}_r^{(N+1)}(O_i) \subseteq \mathcal{N}_r^{(N)}(O_i), \qquad \rho_{N+1}(i; r) \leq \rho_N(i; r). \tag{5}$$

The first inequality is shorthand for every pair (O_i, O_j). These inequalities are the proved relational-shape result. They require no cosmology and no probabilistic assumption. The associated discrete isolation velocity,

$$v_i(N) \equiv D_{N+1}(i) - D_N(i) = \frac{1}{K-1} \sum_{j \neq i} d_{N+1}(o_{i,N+1}, o_{j,N+1}) \geq 0, \tag{6}$$

measures the mean outward distance contributed by the newly appended coordinate.

The cosmological analogy uses a particular probabilistic specialization. Let the coordinates be independent unbiased bits and let d_n be Hamming distance, $d_n(x, y) = \mathbf{1}\{x \neq y\}$. For the exact-match neighborhood $r = 0$, the ensemble-mean density is

$$\bar{\rho}_N \equiv \mathbb{E}[\rho_N(i; 0)] = \Pr[\delta_N(O_i, O_j) = 0] = 2^{-N}, \tag{7}$$

where $\mathbb{E}$ denotes expectation and Pr probability. It is useful to define the effective distinguishing information

$$C_N \equiv -\log_2 \bar{\rho}_N. \tag{8}$$

Here $\log_2$ is the base-two logarithm. Thus $C_N = N$ bits in this special independent-binary model. In general, C_N, the coordinate count N, and the distance increment $v_i(N)$ need not be equal: redundant coordinates can increase N without adding a bit of distinguishing information, while v_i measures distance rather than the logarithmic loss of neighbors. For one unbiased binary coordinate, $C_{N+1} - C_N = 1$ bit whereas the ensemble-mean Hamming contribution is $\mathbb{E}[v_i(N)] = 1/2$.

Now comes the additional dictionary, not a consequence of the theorem. Choose a reference refinement N_* and define inverse density as a dimensionless relational-volume ratio,

$$\frac{V_{\text{rel}}(N)}{V_{\text{rel}}(N_*)} \equiv \frac{\bar{\rho}_{N_*}}{\bar{\rho}_N} = 2^{C_N - C_{N_*}}. \tag{9}$$

Assigning that volume a three-dimensional linear scale gives

$$\frac{a_{\text{rel}}(N)}{a_{\text{rel}}(N_*)} \equiv \left[\frac{V_{\text{rel}}(N)}{V_{\text{rel}}(N_*)}\right]^{1/3} = 2^{(C_N - C_{N_*})/3}. \tag{10}$$

If the discrete sequence C_N admits a continuous interpolation $C(t)$ in cosmic time t, an overdot denotes d/dt, ln is the natural logarithm, and

$$H_{\text{rel}} \equiv \frac{\dot{a}_{\text{rel}}}{a_{\text{rel}}} = \frac{d \ln a_{\text{rel}}}{dt} = \frac{\ln 2}{3}\dot{C}. \tag{11}$$

The earlier N-based formula is the special case $C = N$. The complete logical chain is therefore

$$\boxed{\rho \longrightarrow C = -\log_2 \rho \longrightarrow V_{\text{rel}} \propto 2^C \longrightarrow a_{\text{rel}} \propto 2^{C/3} \longrightarrow H_{\text{rel}} = \frac{\ln 2}{3}\dot{C}.} \tag{12}$$

The mapping can be read directly against the relational formalism (Table 1):

Table 1: Proposed dictionary between the relational-shape quantities and their cosmological readings. The first five rows are internal to the relational geometry; the last three are hypotheses added here.

Relational quantity	Proposed cosmological reading
$O_i^{(N)}$	A candidate state or history specified by N accumulated constraints.
δ_N	Cumulative incompatibility between two candidate histories.
$\mathcal{N}_r^{(N)}(O_i)$	Histories still compatible with O_i within tolerance r.
$\rho_N(i;r)$	Fraction of alternatives remaining locally compatible.
$v_i(N)$	Outward distance added by the next distinguishing constraint.
$C = -\log_2 \bar{\rho}$	Effective independent distinguishing information, in bits.
$V_{\text{rel}} \propto \bar{\rho}^{-1}$	Hypothesized relational volume.
$a_{\text{rel}} \propto V_{\text{rel}}^{1/3}$	Hypothesized three-dimensional scale factor.

Only the first five rows belong to the relational geometry itself. The last three introduce the information measure and then identify its inverse density with a volume and a scale. Identifying a_{rel} with the physical Friedmann–Lemaître–Robertson–Walker (FLRW) scale factor $a(t)$ is the first specifically cosmological hypothesis.

CONCEPTUAL PERSUASIONS

- Adding a new way to tell objects apart can only increase the distance between them, never decrease it. So neighborhoods shrink as detail accumulates. This much is proved.
- If only one object in 2^C still matches you exactly, then C is your distinguishing information, measured in bits.
- The added step: treat the number of remaining matches as a volume—the fewer the matches, the larger the volume—and take its cube root to get a length.
- That length is then identified with the cosmic scale factor. The thinning is a theorem; this identification is an assumption.

The accumulated-history idea

A present state is not specified only by what exists now. In a dynamical world, it must also be compatible with an accumulated past. As the temporal baseline grows, two histories that once appeared equivalent may become distinguishable. The analogy with structural complexity is therefore direct: history supplies additional relational constraints, thinning the set of states that remain locally compatible with the present.

Operationally, fix a comoving domain—a spatial region carried with the mean cosmic flow—and a comparison ensemble of candidate histories. At cosmic time t, $O_i^{(N(t))}$ denotes one history described by the constraints accumulated so far, and coordinate o_{in} records the value of the nth independent relational constraint. The radius r specifies how much cumulative incompatibility is still regarded as locally equivalent. If later times append rather than erase constraints, then for $t_2 > t_1$ the relational theorem gives

$$\mathcal{N}_r^{(N(t_2))}(O_i) \subseteq \mathcal{N}_r^{(N(t_1))}(O_i), \qquad \bar{\rho}(t_2) \leq \bar{\rho}(t_1), \qquad C(t_2) \geq C(t_1). \tag{13}$$

Thus accumulated time is not inserted as a Euclidean distance coordinate. Rather, time indexes a filtration of history neighborhoods, and $C(t)$ counts the effective distinctions revealed by that filtration. This is the direct mathematical connection between accumulated history and structural complexity.

The cleanest scale-free assumption is that each doubling of cosmic age adds a fixed number d_{eff} of effective independent bits,

$$C(t) - C_* = d_{\text{eff}} \log_2(t/t_*), \tag{14}$$

where $t_* > 0$ is a reference cosmic time and $C_* = C(t_*)$. In the independent binary specialization one may write the same equation as $N(t) - N_* = d_{\text{eff}} \log_2(t/t_*)$, where $N_* = N(t_*)$. Substitution into Eqs. (10) and (11), followed by the hypothesis $a = a_{\text{rel}}$, yields

$$\frac{a(t)}{a_*} = \left(\frac{t}{t_*}\right)^p, \qquad H(t) = \frac{p}{t}, \qquad p = \frac{d_{\text{eff}}}{3}. \tag{15}$$

Here $a_* = a(t_*)$, $H \equiv \dot{a}/a$ is the physical Hubble rate, and p is the constant power-law index. The hypothesis has become a definite cosmology. It is also now testable.

There are two closely related ways to interpret "three dimensions plus accumulated time," and it is important not to mix them.

In the first, history increases the information required to maintain a *three-dimensional* state. One new distinction per spatial channel per doubling gives $d_{\text{eff}} = 3$. Then

$$C(t) - C_* = 3 \log_2(t/t_*), \qquad a(t) \propto t, \qquad H(t) = \frac{1}{t}. \tag{16}$$

Time drives the accumulation, but is not itself counted as an interchangeable fourth coordinate.

In the literal version, the three spatial directions and accumulated time are all counted positively, so $d_{\text{eff}} = 4$. This gives

$$C(t) - C_* = 4 \log_2(t/t_*), \qquad a(t) \propto t^{4/3}, \qquad H(t) = \frac{4}{3t}. \tag{17}$$

The distinction matters. Equations (16) and (17) make different predictions and provide two clean benchmarks against which the required evolution of d_{eff} can be measured.

CONCEPTUAL PERSUASIONS

- A present state has to be consistent with everything that already happened. Each past event adds another constraint, so the list of constraints grows with time. Time supplies the coordinates; it is not one of them.
- Two histories that looked identical early on become distinguishable later, once enough record has built up to separate them.
- The assumption: every doubling of the age of the Universe adds the same number of new bits, d_{eff}. No epoch is treated as special.
- That alone gives $a \propto t^p$ with $p = d_{\text{eff}}/3$—a specific expansion history, which data can check.
- Three spatial channels means $d_{\text{eff}} = 3$ and no acceleration. Counting time as a fourth means $d_{\text{eff}} = 4$ and slight acceleration.

Why the present scale looks so promising

Let $t_0 = 13.80\,\mathrm{Gyr}$ denote the measured present age of the Universe, where Gyr means 10^9 years, and let $H_0 \equiv H(t_0)$ be the present Hubble rate. The superscripts (3) and (4) below label the three- and four-channel hypotheses; they are not exponents. The three-channel law gives

$$\begin{aligned} H_0^{(3)} &= \frac{1}{t_0} \\ &= \frac{3.0856776 \times 10^{19}\,\mathrm{km/Mpc}}{13.80 \times 3.15576 \times 10^{16}\,\mathrm{s}} \\ &= 70.85\,\mathrm{km\,s^{-1}\,Mpc^{-1}}. \end{aligned} \tag{18}$$

Here Mpc denotes a megaparsec. This lies 5.1% above the Planck base-ΛCDM value, where ΛCDM denotes the standard cosmology with a cosmological constant Λ and cold dark matter, $67.4 \pm 0.5\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ [1], 3.9% above the DESI DR2+cosmic microwave background (CMB) value used in the accompanying analysis, $68.17\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ [2], and 3.0% below the SH0ES distance-ladder value $73.04 \pm 1.04\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ [3]. Because the CMB-inferred age and H_0 are strongly correlated and derived within ΛCDM itself, Eq. (18) is not an independent prediction of H_0; it is a consistency of scale, not a measurement.

That is more than a dimensional order-of-magnitude match: it is a few-percent match to the scale of the current expansion. In information language, the corresponding present rate is

$$\dot{C}_0 = \frac{3H_0}{\ln 2} = 0.3136\,\mathrm{bits/Gyr},$$

where $\dot{C}_0 = \dot{C}(t_0)$. This is one effective distinction every 3.19 billion years. In the independent-binary specialization, $C = N$ and hence $\dot{C}_0 = \dot{N}_0$.

The literal four-channel law predicts a larger normalization:

$$H_0^{(4)} = \frac{4}{3t_0} = 94.47\,\mathrm{km\,s^{-1}\,Mpc^{-1}}.$$

Thus the especially close numerical connection belongs to the idea that accumulated history increases the distinguishing complexity of a three-dimensional state. Treating time as a fourth Euclidean metadata coordinate gives a distinct, readily testable benchmark.

The successful number also has a conventional cosmological explanation. The dimensionless product $H_0 t_0$ is close to unity in the observed Universe: approximately 0.95 for Planck, 0.96 for DESI DR2+CMB, and 1.03 for SH0ES. Any ansatz $H \sim 1/t$ will therefore land near the current Hubble scale. The real test is whether it predicts the function $H(z)$ and the acceleration, not whether it reproduces one inverse timescale today.

CONCEPTUAL PERSUASIONS

- Put in the measured age, 13.80 Gyr, and the model returns $H_0 = 70.85$, within a few percent of every measurement. Nothing was fitted.
- The implied rate is very slow: about one new distinction per comoving region every 3.2 billion years.
- But $H_0 t_0$ is already close to 1 in the real Universe, so any model of the form $H \sim 1/t$ would land here. The agreement is expected rather than informative.
- The age itself was derived from ΛCDM, so comparing $1/t_0$ with ΛCDM's own H_0 tests consistency, not prediction.
- Matching one epoch says nothing about the rest of the expansion history. That is what the next section tests.

What the simplest construction teaches

For a power-law scale factor $a(t) \propto t^p$, the deceleration parameter is

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = \frac{1}{p} - 1. \tag{19}$$

Here $\dot{a} = da/dt$ and $\ddot{a} = d^2a/dt^2$; negative q denotes accelerated expansion. The three-channel model has $p = 1$ and therefore $q = 0$: it supplies the coasting benchmark. The literal $3+1$ model has $p = 4/3$ and $q = -1/4$, so the same formalism also naturally accesses accelerated expansion. The observed late-time value asks for a stronger and evolving effect. For comparison, the present flat-ΛCDM value for $\Omega_m = 0.3027$ is $q_0 \simeq -0.546$, where Ω_m is the present matter density as a fraction of the critical density and the subscript 0 denotes the present epoch.

The relational bridge itself suggests the generalization. Define the instantaneous effective number of new bits per doubling of cosmic age by

$$d_{\text{eff}}(t) \equiv \frac{dC}{d\log_2 t}. \tag{20}$$

Equation (11) then gives the exact dictionary

$$d_{\text{eff}}(t) = 3H(t)t, \qquad H(t) = \frac{d_{\text{eff}}(t)}{3t}. \tag{21}$$

Consequently,

$$q(t) = -1 - \frac{\dot{H}}{H^2} = -1 + \frac{3}{d_{\text{eff}}(t)} - \frac{3t\dot{d}_{\text{eff}}(t)}{d_{\text{eff}}(t)^2}. \tag{22}$$

The constant-channel models set $\dot{d}_{\text{eff}} = 0$ and recover $q = 3/d_{\text{eff}} - 1 = 1/p - 1$. In the broader relational program, accelerated expansion can depend both on the amount of new distinguishing information and on how its rate changes. The cosmological data can therefore be read as a target curve for $d_{\text{eff}}(t)$ rather than only as a verdict on a fixed power law.

The expansion history supplies the stronger test. Normalize $a(t_0) = a_0 = 1$ and define cosmological redshift by $1 + z = a_0/a(t)$, so $z = 0$ today and $H_0 = H(z = 0)$. Equation (15) predicts

$$H(z) = H_0(1 + z)^{1/p}. \tag{23}$$

Let c be the speed of light and r_d the comoving sound horizon at the baryon-drag epoch. In a spatially flat FLRW geometry, the radial Hubble distance and transverse comoving distance are

$$D_H(z) \equiv \frac{c}{H(z)}, \qquad D_M(z) \equiv c\int_0^z \frac{dz'}{H(z')}. \tag{24}$$

Writing the dimensionless nuisance scale as $X \equiv c/(H_0 r_d)$ gives

$$\frac{D_H(z)}{r_d} = X(1 + z)^{-1/p}, \tag{25}$$

$$\frac{D_M(z)}{r_d} = X\int_0^z (1 + z')^{-1/p}\, dz'. \tag{26}$$

For $p = 1$, the second expression is $X \ln(1 + z)$. For $p \neq 1$ it is

$$\frac{D_M(z)}{r_d} = X\frac{(1 + z)^{1-1/p} - 1}{1 - 1/p}. \tag{27}$$

These are the complete distance–redshift predictions of the construction.

We compare them with the published 13-element DESI DR2 BAO vector and full covariance [2]. The nuisance scale X is fitted analytically for every row, so the result isolates the redshift-dependent shape independently of a model's chosen H_0 or sound horizon. Writing the data vector as $\mathbf{y}$, a model prediction as $\mathbf{m}$, and the published covariance matrix as $\mathbf{\Sigma}_{\text{BAO}}$, the reported statistic is

$$\chi^2 = (\mathbf{y} - \mathbf{m})^\mathsf{T} \mathbf{\Sigma}_{\text{BAO}}^{-1} (\mathbf{y} - \mathbf{m}), \tag{28}$$

where T denotes transpose. The degrees of freedom equal the number of data values minus the number of fitted parameters. The matter-like $p = 2/3$ row of Table 2 is included as a familiar power-law benchmark.

Table 2: Fits to the 13-element DESI DR2 BAO vector with the nuisance scale X profiled analytically for every row. Fixed power laws are compared with the best single power and with flat ΛCDM.

History law	χ^2	degrees of freedom
Matter-like, $p = 2/3$	1420.18	12
Three spatial channels, $p = 1$	187.09	12
Literal 3D plus time, $p = 4/3$	1364.93	12
Best constant power, $p = 0.9125$	96.97	11
Flat ΛCDM, $\Omega_m = 0.3027$	10.63	12

The comparison gives a precise design constraint. The three-channel law retains its striking present-day normalization, while the distance data show that a constant information rate per logarithmic time cannot capture the full redshift dependence. Literal $3+1$ counting probes a different constant slope and is still farther from the measured shape. Even the best constant p cannot follow all epochs. Fitting the free power law only below $z = 1$ gives $p = 1.1185$, but its six genuinely held-out predictions at $z \geq 1$ have $\chi^2 = 670.72$. This is useful predictive information: the next construction must promote d_{eff} to a dynamical quantity, or replace total accumulated complexity by a conditional information rate.

CONCEPTUAL PERSUASIONS

- A constant d_{eff} fixes the deceleration for all time: $p = 1$ gives none, $p = 4/3$ gives a little. The real Universe accelerates more than either.
- Because $d_{\text{eff}}(t) = 3H(t)t$, any measured expansion rate converts directly into a bit rate. The data can be read in the model's own terms.
- Acceleration can then come from two places: the size of d_{eff}, or its rate of change. A fixed law only has the first.
- No constant power fits the DESI distances. Fitting p below $z = 1$ and predicting the six points above it gives $\chi^2 = 670.72$—a clear failure of a real prediction.
- The failure is specific rather than fatal: it says d_{eff} must change with time.

Toward a relativistic information rate

Time is not simply another spatial coordinate. Proper time depends on the worldline, causal order distinguishes past from future, and the spacetime metric has Lorentzian rather than Euclidean signature. Information-based approaches that take spacetime seriously therefore work with local horizons or causal order rather than appending a positive time distance [4, 5].

A natural relativistic extension is the four-volume of an observer's causal past in flat FLRW spacetime,

$$V_4(\tau_o) = \frac{4\pi}{3} \int_0^{\tau_o} c\, d\tau'\, a^3(\tau') \left[\int_{\tau'}^{\tau_o} \frac{c\, d\tau''}{a(\tau'')} \right]^3 . \tag{29}$$

Here τ_o is the observer's proper time, τ' and τ'' are integration variables along the past history, $a(\tau)$ is the FLRW scale factor, and c is retained so V_4 has dimensions of length to the fourth power. The observer has unit timelike four-velocity u^μ, ∇_μ is the spacetime covariant derivative, and repeated Greek indices run over spacetime coordinates. The covariant expansion scalar is $\theta \equiv \nabla_\mu u^\mu = 3H$ for the comoving FLRW flow.

To test a constitutive connection, define either a raw causal-history variable $\mathcal{C}_{\text{raw}} = V_4$ or the dimensionless logarithmic variable

$$\mathcal{C}_{\text{log}}(\tau_o) \equiv \log_2 \left[\frac{V_4(\tau_o)}{V_{4,*}} \right], \tag{30}$$

where $V_{4,*} > 0$ is an arbitrary reference four-volume. If causal history directly generated expansion with a constant constitutive coupling, one would write

$$3H = \kappa \dot{\mathcal{C}}, \tag{31}$$

where the overdot here is $d/d\tau_o$ and κ is the coupling between the chosen causal-history variable and expansion. Its units depend on whether $\mathcal{C}$ is raw or logarithmic. For the dimensionless bit count of the direct relational dictionary, Eq. (11) would give $\kappa = \ln 2$.

Evaluated from $z = 2.33$ to the present along the DESI-calibrated background, the coupling required for raw V_4 changes by a factor of 321.8. Taking the logarithm removes most of the drift, but the required coupling still changes by a factor of 1.514, with present value 0.5373 rather than the $\ln 2 = 0.6931$ suggested by the direct neighborhood dictionary. This substantial improvement from raw volume to logarithmic volume is encouraging: it shows that information-like normalization is the right direction, while also indicating that total causal volume alone is not yet the desired invariant.

The result points to a natural refinement. Total causal volume counts shared history again and again, whereas the relational-shape construction responds to new distinctions. A more faithful covariant quantity would therefore be the conditional new information between neighboring causal slices after their common past has been removed. That extension preserves the core relational idea while replacing accumulated amount by an independent information rate.

CONCEPTUAL PERSUASIONS

- Time cannot be swapped for a spatial direction: elapsed time depends on the path taken, and causal order separates past from future. A covariant version has to respect that.
- The relativistic version of "accumulated history" is the four-volume of the observer's causal past—everything that could have influenced them.
- Using that volume directly fails: the coupling it requires changes by a factor of 322 since $z = 2.33$. Using its logarithm cuts the change to 1.5.
- The drift that remains has a clear cause. The four-volume contains the same early history at every step, so old information gets counted again and again.
- The fix is to count only what is new: the information one causal slice adds beyond the one before it.

The constructive next step

The cosmological analogy makes quantitative contact immediately. The exact neighborhood law produces a transparent dictionary between distinction rate and expansion rate, and the measured age of the Universe gives the correct scale for H_0. The distance–redshift curve then adds something valuable: it shows that the relevant independent-information rate is dynamical rather than a fixed channel count.

This leaves a concrete research program. Accumulated history can be analogous to increased structural complexity, with $d_{\text{eff}}(t) = dC/d\log_2 t = 3H(t)t$ providing the observational target. The next step is to define C as an independent, covariant conditional-information measure on neighboring causal slices and derive its dynamics. Such a measure would preserve the neighborhood-thinning mechanism while respecting the special role of time and avoiding repeated counts of shared causal history.

The near equality $H_0 \simeq 1/t_0$ remains an elegant clue about why the idea is quantitatively compelling: by Eq. (21), it says $d_{\text{eff}}(t_0) \simeq 3$. The simple model has therefore identified both the natural present scale and the mathematical quantity whose evolution must be explained. That is a promising starting point for a relational account of cosmic expansion.

CONCEPTUAL PERSUASIONS

- What works is the chain from bit rate to expansion rate. It reproduces today's Hubble scale without being given it.
- What fails is the assumption that d_{eff} is constant. It has to become a dynamical quantity.
- Translated, $H_0 \simeq 1/t_0$ says only that $d_{\text{eff}} \simeq 3$ at present. It does not say three channels at every epoch.
- The remaining work is well defined: build C as the conditional information between neighboring causal slices, derive how it evolves, and test it against $d_{\text{eff}}(t) = 3H(t)t$, which the data already give.

References

[1] Planck Collaboration. Planck 2018 results. vi. cosmological parameters. *Astronomy & Astrophysics*, 641:A6, 2020. doi: 10.1051/0004-6361/201833910.

[2] DESI Collaboration. DESI DR2 Results II: Measurements of baryon acoustic oscillations and cosmological constraints. 2025.

[3] Adam G. Riess et al. A comprehensive measurement of the local value of the hubble constant with 1 km/s/mpc uncertainty from the hubble space telescope and the SH0ES team. *Astrophysical Journal Letters*, 934:L7, 2022. doi: 10.3847/2041-8213/ac5c5b.

[4] Ted Jacobson. Thermodynamics of spacetime: The einstein equation of state. *Physical Review Letters*, 75:1260–1263, 1995. doi: 10.1103/PhysRevLett.75.1260.

[5] Sumati Surya. The causal set approach to quantum gravity. *Living Reviews in Relativity*, 22:5, 2019. doi: 10.1007/s41114-019-0023-1.