Magnetic Fields from Hawking-Radiated Charged Particles vs. Accretion Disk Dynamo in Rotating Kerr Black Holes

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Abstract

We present a theoretical comparison of two mechanisms for magnetic field generation in the vicinity of a rotating (Kerr) black hole: (1) charged-particle emission via Hawking radiation, and (2) magnetohydrodynamic (MHD) dynamo action in the accretion disk (Blandford–Znajek process). We derive closed-form expressions for the induced field strengths, analyze their dependence on black hole mass M and spin parameter a, and identify the unphysical rotation rate required for parity of the two fields. Despite the formal superlinear scaling of Hawking-induced magnetism with spin, we show it remains negligible for all astrophysical black holes. This work highlights a curious intersection of quantum and classical effects, suggesting potential—but practically unobservable—signatures of quantum processes in extreme gravitational environments.

1 Introduction

Rotation and conducting media in astrophysics often conspire to produce significant magnetic fields: Earth via its liquid iron convective core, and black holes via relativistic plasmas in accretion disks. A curious question arises: can quantum evaporation—Hawking radiation—of charged particles around a Kerr black hole generate a magnetic field competitive with the classical MHD dynamo? We sketch a paper detailing the mathematical framework and scaling analysis to resolve this question. Connecting these regimes sheds light on where quantum gravity effects might subtly influence observable astrophysical phenomena.

2 Preliminaries and Notation

- Gravitational constant: G, speed of light: c, vacuum permeability: μ_0.
- Black hole mass M, angular momentum per unit mass $a = J/M$, horizon radius

$$r_+ = \frac{GM}{c^2} + \sqrt{\left(\frac{GM}{c^2}\right)^2 - a^2}.$$

- Horizon angular velocity:

$$\Omega_H = \frac{ac}{2GMr_+}.$$

- Dimensionless spin: $a_* = \frac{a}{GM/c} \in [0,1]$.

These definitions align with standard GR notation and ensure clarity when comparing to dynamo theory in planetary contexts.

3 Hawking Radiation–Induced Magnetic Field

3.1 Hawking Temperature and Particle Flux

Temperature of a Kerr black hole is

$$T_H = \frac{\hbar c^3}{8\pi GMk_B}\frac{\sqrt{1-a_*^2}}{1+\sqrt{1-a_*^2}}.$$

Total power radiated to all species:

$$P_H \sim \frac{\hbar c^6}{G^2M^2}(1-a_*^2)^{1/2}.$$

Assuming mean particle energy $\langle E\rangle \approx k_BT_H$, emission rate

$$\dot{N} \sim \frac{P_H}{k_BT_H} \approx \frac{c^3}{GM}(1-a_*^2)^{-1/2}.$$

This relation reveals that moderate rotation enhances Hawking flux, though near-extremality suppression limits the effect.

3.2 Charged-Particle Current Loop

Let fraction $f = n_c/N$ be the charged-out fraction and $q = e$ the elementary charge. Net current at radius r_+:

$$I(\Omega_H) = fq\dot{N}(\Omega_H) = fe\frac{c^3}{GM}(1-a_*^2)^{-1/2}.$$

Magnetic dipole moment:

$$\mu_{\rm rad} = I\pi r_+^2 = fe\frac{\pi c^3}{GM}(1-a_*^2)^{-1/2}r_+^2.$$

Field on axis near r_+:

$$B_{\rm rad}(\Omega_H) = \frac{\mu_0}{4\pi}\frac{2\mu_{\rm rad}}{r_+^3} = \frac{\mu_0 fec^3}{2GM}(1-a_*^2)^{-1/2}\frac{1}{r_+}.$$

Expanding $r_+ \approx 2GM/c^2$ yields the scaling

$$B_{\rm rad} \sim \frac{\mu_0 fec^5}{2G^2M^2}(1-a_*^2)^{-1/2}.$$

Physically, this shows how quantum processes could in principle seed magnetic fields, though the magnitude is far smaller than MHD counterparts.

4 Plasma Dynamo–Induced Magnetic Field

4.1 Blandford–Znajek Scaling

The magnetospheric jet power is

$$P_{\rm BZ} \approx \frac{\kappa}{4\pi}\Phi_B^2\Omega_H^2,$$

with magnetic flux Φ_B through the horizon. Near the hole,

$$B_{\rm plasma} \approx \frac{\Phi_B}{\pi r_+^2} \propto \Omega_H \quad \Rightarrow \quad B_{\rm plasma}(a_*) \sim B_0 a_*.$$

Empirical estimates give $B_0 \sim 10^8$ G for $M = 10M_\odot$. This linear scaling underscores the direct role of frame dragging in amplifying MHD fields.

5 Theorem: Absence of Physical Spin Equality

Theorem 1. For any astrophysical Kerr black hole with dimensionless spin $a_* \in [0,1]$, there is no physically admissible spin a_* such that $B_{\rm rad}(a_*) = B_{\rm plasma}(a_*)$.

Proof. From Sections 3 and 4,

$$B_{\rm rad}(a_*) = \frac{\mu_0 f e c^5}{2G^2M^2}(1-a_*^2)^{-1/2}, \quad B_{\rm plasma}(a_*) = B_0 a_*.$$

Define the dimensionless constant

$$C \equiv \frac{\mu_0 f e c^5}{2G^2M^2B_0} \sim 10^{-65}$$

(for $M = 10M_\odot$, $f = 0.1$, $B_0 \sim 10^8$ G). Equating fields gives

$$C(1-a_*^2)^{-1/2} = a_* \quad \Rightarrow \quad a_*(1-a_*^2)^{1/2} = C.$$

On $0 < a_* \leq 1$, the function

$$g(a_*) = a_*(1-a_*^2)^{1/2}$$

attains its maximum value $1/2$ at $a_* = 1/\sqrt{2}$ and is strictly less elsewhere. Since $C \ll 1/2$, the only formal solution is

$$a_* \approx C \sim 10^{-65},$$

which lies far below any astrophysically observed spin ($a_* \gtrsim 0.1$). Thus, no physical $a_* \in [0,1]$ satisfies $B_{\rm rad} = B_{\rm plasma}$. ■

6 Discussion

- Spin scaling: Hawking-induced field formally diverges as $a_* \to 1$ via $(1-a_*^2)^{-1/2}$, but quantum backreaction and greybody factors moderate this near-extremality.
- Astrophysical relevance: Even at maximal spin, Hawking magnetism remains 10^{65} times weaker than plasma dynamos—a gulf analogous to comparing thermal noise with cosmic jet power.
- Extensions: In hypothetical micro black holes ($M \sim 10^{11}$ kg), parity could be approached, motivating analogue gravity experiments in the lab. This invites exploration of sonic-black-hole systems to probe quantum-field-induced magnetism.

7 Conclusion

Hawking-radiation-driven magnetism, though conceptually intriguing, is completely overshadowed by classical plasma dynamos in realistic Kerr black holes. Our analysis sets a clear boundary between quantum evaporation effects and astrophysical MHD, guiding future theoretical and analogue studies.

References

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