Energy and Identity in Atomic Photon Absorption

Energy must localize in the atom; identity need not

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Abstract. Absorption globally accounts for both energy and identity. The photon's energy is conserved in the full output and, up to recoil, localized in the atom; the photon's *identity*, the information distinguishing which photon was absorbed, is conserved as reference correlation across the enlarged output $A'D$. The difference is localization. Energy's retained fraction is pinned near unity by the recoil scale $\hbar\omega/2Mc^2 \sim 10^{-9}$ for an optical photon on hydrogen, while the identity localization fraction $\eta_{\text{abs}} = I(R{:}A')_{\rho^+}/I(R{:}A\gamma)_{\rho^-}$ is dynamical over $[0,1]$. The contribution is to place this retained-correlation quantity in direct contrast with energy: same global conservation law, different localization law. We make this precise, compute η_{abs} in a two-state model, and show that internal localization is capped by dimension, $\eta_{\text{int}} \leq \log_2 d_e/\mathcal{J}_{\text{abs}}$. The swept-overlap curve is the falsifiable prediction; the $\log_2 d_e/\log_2 N$ envelope is the corresponding finite-carrier no-go.

Keywords: photon absorption; mutual information; identity localization; quantum memory; atomic state transfer.

1 Setup

Consider a hydrogen atom A absorbing a photon γ. Here A denotes the atomic subsystem (electron and nucleus) and γ the photon subsystem.

Before absorption, the relevant physical system is the composite

$$S_- = A\gamma,$$

where S_- is the **pre-absorption system**.

After absorption, the photon is no longer an independent subsystem in that absorbing mode. The relevant system is the excited atom,

$$S_+ = A'.$$

Here S_+ is the **post-absorption system**; the prime marks the same atom after the transition.

Ordinary atomic physics writes the transition schematically as

$$|g\rangle_A\,|1\rangle_\gamma \;\longrightarrow\; |e\rangle_A\,|0\rangle_\gamma,$$

where $|g\rangle_A$ is the initial atomic state, $|e\rangle_A$ the excited state, and $|0\rangle_\gamma$ the emptied absorbing mode.

The question posed here is a different one:

How much of the globally conserved atom–photon identity localizes in the post-absorption atom?

That is not a new conservation law. It is a localization question, posed in parallel with energy.

Thesis. Across the full output $A'D$, energy and identity obey parallel global conservation laws. They diverge locally: energy retention is recoil-pinned at $1 - O(10^{-9})$, while the identity localization fraction η_{abs} is dynamical over $[0, 1]$.

2 Energy versus identity

Two quantities cross the absorption boundary. Globally they behave alike: track every output degree of freedom and neither energy nor identity is lost. Locally they diverge.

Energy is conserved and additive; absorption is *defined* by where it goes. The photon's $\hbar\omega$ becomes internal atomic excitation, while a vanishing share escapes as center-of-mass recoil, a fraction of order $\hbar\omega/2Mc^2 \sim 10^{-9}$ for an optical photon and an atomic mass M, so the energy retained by the atom is pinned at essentially unity. Energy has a destination, and the atom is it.

Identity, the information distinguishing which photon was absorbed, is operationalized below as the correlation $I(R{:}A')$ between the atom and a reference R that labels the incoming alternatives. Like energy, its total is conserved across the full post-absorption system (§4): nothing is destroyed if everything is tracked. Unlike energy, its destination is not fixed. The localization fraction in the atom alone,

$$\eta_{\text{abs}} = \frac{I(R{:}A')_{\rho^+}}{I(R{:}A\gamma)_{\rho^-}} \in [0, 1],$$

is dynamical, not a near-constant. Both quantities can shed into discarded modes D: recoil for energy, decohered correlations for identity. But energy's leak is mass-suppressed, while identity's can be *everything*. In quantum-memory language, η_{abs} coincides mechanically with the retained-correlation-with-a-reference storage figure of merit [5, 6, 7].

That overlap is intentional. The coherent-information / mutual-information lineage asks how much reference correlation a device preserves; memory engineering wants $\eta_{\text{abs}} \to 1$. Here the same quantity is asked a different question: does a globally conserved identity localize the way energy does? The contribution is the contrast. Energy and identity share a global conservation law, but energy localization is recoil-pinned while identity localization is free over $[0, 1]$. Thus $\mathcal{J}_{\text{abs}}$ is not I_e with a new name; it is the same conserved reference-correlation quantity placed against energy's recoil-pinned localization. The asymmetry is not merely that a correlation fails to localize like a local observable; it is that two quantities share one global accounting law while one is pinned to $1 - O(10^{-9})$ and the other ranges freely.

	conserved across $A'D$	localization in A'
energy	yes	≈ 1 (recoil-limited)
identity $I(R{:}A')$	yes	$\eta_{\text{abs}} \in [0, 1]$

The machinery below is standard quantum information; the claim it serves is the asymmetry above.

3 Reference system

Introduce a reference system R that stores, labels, or purifies the relevant alternatives of the original atom–photon system.

The identity functional is

$$\mathcal{I}_S = I(R{:}S),$$

where $\mathcal{I}_S$ is the identity content of subsystem S and $I(R{:}S)$ is mutual information with the reference [1, 2]. This is the standard way to track how much quantum information survives a process [5]:

$$I(R{:}S) = S(\rho_R) + S(\rho_S) - S(\rho_{RS}).$$

Here ρ_R, ρ_S, and ρ_{RS} are the reduced and joint density matrices. The entropy is

$$S(\rho) = -\operatorname{Tr}\bigl(\rho \log_2 \rho\bigr),$$

with $\log_2$ fixing the unit as the **bit**. Thus $I(R{:}S)$ is the number of reference bits recoverable from S.

For absorption, the initial subsystem is $S_- = A\gamma$ and the final subsystem of interest is $S_+ = A'$.

4 Full absorption map

After absorption, some degrees of freedom may lie outside the final atom. Call them D: everything not included in A', including recoil, unobserved field modes, environmental modes, the empty photon mode, or other discarded correlations. The closed-system process is

$$A\gamma \;\longrightarrow\; A'D,$$

where $A'D$ is the final atom plus discarded degrees of freedom.

Let $\rho^-_{RA\gamma}$ be the pre-absorption state, and let absorption act by an isometry

$$V_{\text{abs}} : \mathcal{H}_{A\gamma} \longrightarrow \mathcal{H}_{A'D},$$

from the initial atom–photon Hilbert space to the output space. An isometry preserves inner products while allowing extra output degrees of freedom; it is the unitary dilation of any physical absorption channel [5, 1].

The post-absorption state is then

$$\rho^+_{RA'D} = \bigl(\mathrm{Id}_R \otimes V_{\text{abs}}\bigr)\, \rho^-_{RA\gamma}\, \bigl(\mathrm{Id}_R \otimes V^\dagger_{\text{abs}}\bigr),$$

where $+$ means "after absorption," Id_R is the identity on R, and $V^\dagger_{\text{abs}}$ is the adjoint of V_{abs}.

Because mutual information is invariant under local isometries on the non-reference system [2],

$$I(R{:}A\gamma)_{\rho^-} = I(R{:}A'D)_{\rho^+}.$$

Defining the total identity content of the initial atom–photon system relative to R,

$$\mathcal{J}_{\text{abs}} := I(R{:}A\gamma)_{\rho^-},$$

we obtain the basic cross-boundary absorption invariant:

$$\mathcal{J}_{\text{abs}} = I(R{:}A\gamma)_{\rho^-} = I(R{:}A'D)_{\rho^+}.$$

The total initial atom–photon identity is preserved in the enlarged output $A'D$. This is the identity analog of energy conservation. Energy and identity diverge only at the next step: how much of the conserved total localizes in the atom alone?

5 Identity retained by the atom

The question is not whether information survives in $A'D$, but how much survives in the atom *alone*. Define the retained atomic identity

$$\mathcal{I}_{\text{ret}} := I(R{:}A')_{\rho^+},$$

the amount of pre-absorption atom–photon identity recoverable from the post-absorption atom alone.

The chain rule for mutual information gives

$$I(R{:}A'D) = I(R{:}A') + I(R{:}D \mid A'),$$

where $I(R{:}D \mid A')$ is the reference information still in D once A' is known. Therefore

$$I(R{:}A\gamma)_{\rho^-} = I(R{:}A')_{\rho^+} + I(R{:}D \mid A')_{\rho^+}.$$

Defining the leakage term

$$\mathcal{L}_{\text{abs}} := I(R{:}D \mid A')_{\rho^+},$$

the identity not captured by the final atom, the central decomposition is

$$\mathcal{J}_{\text{abs}} = \mathcal{I}_{\text{ret}} + \mathcal{L}_{\text{abs}}, \qquad \text{i.e.} \qquad I(R{:}A\gamma)_{\rho^-} = I(R{:}A')_{\rho^+} + I(R{:}D \mid A')_{\rho^+}.$$

The initial identity ($\mathcal{J}_{\text{abs}}$) splits into the part retained in the excited atom ($\mathcal{I}_{\text{ret}}$) and the part leaked outside it ($\mathcal{L}_{\text{abs}}$). Nonnegativity of conditional mutual information, strong subadditivity [4], gives

$$I(R{:}A')_{\rho^+} \le I(R{:}A\gamma)_{\rho^-}.$$

The final atom cannot contain more identity than the original atom–photon system: tracing out D cannot increase correlation with the reference [5, 2].

6 Localization fraction

Define the normalized absorption localization fraction

$$\eta_{\text{abs}} := \frac{I(R{:}A')_{\rho^+}}{I(R{:}A\gamma)_{\rho^-}} = \frac{\mathcal{I}_{\text{ret}}}{\mathcal{J}_{\text{abs}}},$$

the share of initial identity localizing in the final atom. Equivalently, this is the same retained-correlation ratio, used here as a localization fraction for the absorption map. Energy's localization fraction is pinned at ≈ 1; identity is free to range. Assuming $I(R{:}A\gamma)_{\rho^-} > 0$,

$$0 \le \eta_{\text{abs}} \le 1.$$

The limits are $\eta_{\text{abs}} = 1$ when all identity-relevant information is recoverable from the final atom alone, and $\eta_{\text{abs}} = 0$ when the atom absorbs the photon's energy while retaining none of the reference-distinguishing information. Using the leakage term,

$$\eta_{\text{abs}} = 1 - \frac{\mathcal{L}_{\text{abs}}}{\mathcal{J}_{\text{abs}}},$$

so the localization fraction is one minus fractional identity leakage.

7 Classical reference model

Let R store a classical label x with probabilities p_x, where x may label polarization, frequency, wavepacket shape, angular momentum, or arrival time. Before absorption, take

$$\rho^-_{RA\gamma} = \sum_x p_x \, |x\rangle\langle x|_R \otimes |g\rangle\langle g|_A \otimes |1_x\rangle\langle 1_x|_\gamma,$$

where $|x\rangle_R$ is the reference state, $|g\rangle_A$ the initial atom, and $|1_x\rangle_\gamma$ the one-photon state. Absorption sends

$$|g\rangle_A \, |1_x\rangle_\gamma \;\longrightarrow\; \rho^x_{A'D},$$

the final joint state conditioned on x. The final atom state is

$$\rho^x_{A'} = \mathrm{Tr}_D\big[\rho^x_{A'D}\big],$$

with Tr_D the partial trace over D. Hence

$$\rho^+_{RA'} = \sum_x p_x \, |x\rangle\langle x|_R \otimes \rho^x_{A'},$$

and

$$I(R{:}A')_{\rho^+} = S\Big(\sum_x p_x \, \rho^x_{A'}\Big) - \sum_x p_x \, S\big(\rho^x_{A'}\big).$$

This is the Holevo information [3] of the ensemble $\{p_x, \rho^x_{A'}\}$. Thus

$$I(R{:}A')_{\rho^+} \;=\; \text{distinguishability of the final atomic states produced by different photon alternatives.}$$

If all final atom states are identical, $I(R{:}A')_{\rho^+} = 0$: the atom absorbed energy, but no photon-label information survived. If they are perfectly distinguishable, the atom retains the full classical identity label. Hence

$$\text{energy absorption} \;\neq\; \text{identity-information absorption.}$$

8 Two-state toy model

Take a one-bit classical reference, $x \in \{0, 1\}$, with $p_0 = p_1 = \frac{1}{2}$. Before absorption,

$$\rho^-_{RA\gamma} = \tfrac{1}{2}\,|0\rangle\langle 0|_R \otimes |g, 1_0\rangle\langle g, 1_0| + \tfrac{1}{2}\,|1\rangle\langle 1|_R \otimes |g, 1_1\rangle\langle g, 1_1|,$$

where $|g, 1_i\rangle = |g\rangle_A |1_i\rangle_\gamma$. Suppose absorption gives

$$|g\rangle_A |1_0\rangle_\gamma \longrightarrow |e_0\rangle_{A'} |d_0\rangle_D, \qquad |g\rangle_A |1_1\rangle_\gamma \longrightarrow |e_1\rangle_{A'} |d_1\rangle_D,$$

with $|e_i\rangle_{A'}$ the final atomic states and $|d_i\rangle_D$ the discarded states.[1] The final atom–reference state is

$$\rho^+_{RA'} = \tfrac{1}{2}\,|0\rangle\langle 0|_R \otimes |e_0\rangle\langle e_0| + \tfrac{1}{2}\,|1\rangle\langle 1|_R \otimes |e_1\rangle\langle e_1|.$$

[1] Because the incoming alternatives are orthogonal ($\langle 1_0 \,|\, 1_1\rangle = 0$, the "one full classical bit" assumed below), isometry of $V_{\rm abs}$ requires the outputs to remain orthogonal: $\langle e_0 \,|\, e_1\rangle \langle d_0 \,|\, d_1\rangle = 0$. Hence whenever the atomic overlap is nonzero, $c_A = |\langle e_0 \,|\, e_1\rangle| > 0$, the discarded modes must satisfy $\langle d_0 \,|\, d_1\rangle = 0$: the environment D carries exactly the distinguishing information the atom fails to retain. This constraint does not affect the reduced atomic state, which depends only on $|e_0\rangle$ and $|e_1\rangle$.

Let the atomic overlap be

$$c_A := \big|\langle e_0 \,|\, e_1 \rangle\big|, \qquad 0 \le c_A \le 1,$$

so $c_A = 0$ means perfectly distinguishable and $c_A = 1$ identical up to phase. The final atom state is

$$\rho_{A'} = \tfrac{1}{2}\,|e_0\rangle\langle e_0| + \tfrac{1}{2}\,|e_1\rangle\langle e_1|,$$

with eigenvalues

$$\lambda_\pm = \frac{1 \pm c_A}{2}.$$

Therefore

$$I(R{:}A')_{\rho^+} = S(\rho_{A'}) = h_2\!\left(\frac{1 + c_A}{2}\right), \qquad h_2(q) = -q \log_2 q - (1-q)\log_2(1-q),$$

with h_2 the binary entropy. If the alternatives carried one full classical bit, then $I(R{:}A\gamma)_{\rho^-} = 1$, and

$$\boxed{\eta_{\rm abs} = h_2\!\left(\frac{1 + c_A}{2}\right).}$$

The limits are clean. Orthogonal outputs give $c_A = 0$ and

$$\eta_{\rm abs} = h_2\big(\tfrac{1}{2}\big) = 1,$$

so the atom fully retains the one bit of photon identity. Identical outputs give $c_A = 1$ and

$$\eta_{\rm abs} = h_2(1) = 0,$$

so the atom absorbed the photon's energy but carries no information about which alternative arrived:

$$\boxed{\text{the atom can absorb energy while erasing the photon's identity label.}}$$

9 Quantum reference version

The reference may also be quantum, entangled with the incoming photon:

$$|\Psi^-\rangle_{RA\gamma} = \alpha\,|0\rangle_R |g\rangle_A |1_0\rangle_\gamma + \beta\,|1\rangle_R |g\rangle_A |1_1\rangle_\gamma, \qquad |\alpha|^2 + |\beta|^2 = 1,$$

with $|\alpha|^2 + |\beta|^2 = 1$. After absorption,

$$|\Psi^+\rangle_{RA'D} = \alpha\,|0\rangle_R |e_0\rangle_{A'} |d_0\rangle_D + \beta\,|1\rangle_R |e_1\rangle_{A'} |d_1\rangle_D.$$

The retained identity is still $\mathcal{I}_{\rm ret} = I(R{:}A')_{\rho^+}$. If A' alone preserves the quantum correlation with R, then

$$I(R{:}A') = 2\,S(\rho_R),$$

with $S(\rho_R)$ the reduced reference entropy. For a maximally entangled qubit reference, $S(\rho_R) = 1$, so

$$I(R{:}A') = 2 \text{ bits}.$$

These are mutual-information bits, not necessarily ordinary classical bits: quantum mutual information counts total correlation, including the quantum part.

10 First proposition

Proposition 1. *Let R be a fixed reference system, and let the pre-absorption subsystem be $S_- = A\gamma$. Suppose absorption is represented by an isometry $V_{\text{abs}} : A\gamma \to A'D$. Then*

$$I(R{:}A\gamma)_{\rho^-} = I(R{:}A'D)_{\rho^+},$$

and moreover

$$I(R{:}A\gamma)_{\rho^-} = I(R{:}A')_{\rho^+} + I(R{:}D \mid A')_{\rho^+},$$

so that

$$I(R{:}A')_{\rho^+} \le I(R{:}A\gamma)_{\rho^-}.$$

The retained, leaked, and normalized quantities are

$$\mathcal{I}_{\text{ret}} = I(R{:}A')_{\rho^+}, \qquad \mathcal{L}_{\text{abs}} = I(R{:}D \mid A')_{\rho^+}, \qquad \eta_{\text{abs}} = \frac{\mathcal{I}_{\text{ret}}}{\mathcal{J}_{\text{abs}}} = \frac{I(R{:}A')_{\rho^+}}{I(R{:}A\gamma)_{\rho^-}}.$$

11 What this says physically

Standard atomic theory measures

$$|g\rangle_A|1\rangle_\gamma \longrightarrow |e\rangle_A|0\rangle_\gamma.$$

This framework measures continuity across it,

$$I(R{:}A\gamma)_{\rho^-} \;\longrightarrow\; I(R{:}A')_{\rho^+} + I(R{:}D \mid A')_{\rho^+}.$$

The photon disappears as an independent subsystem in *every* absorption event, at every η_{abs}. That is the backdrop, not the variable. The variable is the retained identity

$$\mathcal{I}_{\text{ret}} = I(R{:}A')_{\rho^+},$$

the pre-absorption identity recoverable from the post-absorption atom alone.

This is where energy and identity part company. Absorption deposits the photon's energy in the atom: the excitation localizes in A', and energy is conserved. For identity only the *global* total is conserved,

$$I(R{:}A\gamma)_{\rho^-} = I(R{:}A'D)_{\rho^+},$$

the isometry invariant of §4. The accessible quantity, identity retained in the atom alone, is *not* conserved:

$$0 \le \mathcal{I}_{\text{ret}} \le \mathcal{J}_{\text{abs}}, \qquad \mathcal{J}_{\text{abs}} - \mathcal{I}_{\text{ret}} = \mathcal{L}_{\text{abs}} = I(R{:}D \mid A')_{\rho^+},$$

the deficit being shed into D. Energy is conserved and lands in the atom; identity, from the atom alone, need not survive. That asymmetry, not the emptying of the photon mode, is the content of the framework.

This fixes the sharp meaning of the photon being *gone*: none of the photon's distinguishing information survives in the atom,

$$\eta_{\text{abs}} \to 0 \quad \Longleftrightarrow \quad I(R{:}A')_{\rho^+} = 0 \quad \Longleftrightarrow \quad \rho^x_{A'} = \rho^{x'}_{A'} \text{ for all } x, x'.$$

At this endpoint the atom is excited, yet every incoming alternative drives it to the same final state: the atom has forgotten which photon excited it. This is a definition, not a theorem. Its value is to separate "the energy arrived" from "the identity is gone," the latter governed by $\mathcal{J}_{\text{abs}} = \mathcal{I}_{\text{ret}} + \mathcal{L}_{\text{abs}}$ and its endpoint

$$\text{“the photon's identity is gone”} \quad \Longleftrightarrow \quad \eta_{\text{abs}} \to 0 \quad \Longleftrightarrow \quad \mathcal{I}_{\text{ret}} \to 0.$$

12 Two senses of being gone

The note uses *gone* in two registers; the payoff is keeping them apart.

The first is **occupation**. The absorbed mode has photon-number operator $\hat{N}_\gamma = a^\dagger a$; before absorption it is in $|1\rangle$, after in $|0\rangle$. Every observer agrees the mode lost its quantum. Call this *occupation-gone*: exact, universal, and true at every value of $\eta_{\rm abs}$.

The second is **identity**. The photon's distinguishing information leaves the atom only at $\eta_{\rm abs} \to 0$, and only relative to an observer who can read A' but not D. Call this *identity-gone*. The two are independent: a perfect quantum memory [6, 7] is occupation-gone with $\eta_{\rm abs} = 1$: the mode empties while the atom keeps the full record.

They separate because they rest on different conserved-quantity structures. Occupation-gone is licensed by the *non*-conservation of photon number: the light–matter coupling does not commute with $\hat{N}_\gamma$, so the photon is a losable object. The electron and nucleus are opposite: their conserved, superselected charges persist across the transition. Number is *rigid*: a carrier either persists or can vanish, observer-independently. Identity is *soft*: the mutual information $I(R{:}\,\cdot\,)$ is conserved only globally (§4), and its localization in a chosen factor can range across $[0, \mathcal{J}_{\rm abs}]$ (§5). Thus "the photon is gone" is a hard number fact, while "the photon's identity is gone" is an access-relative information fact [11].

$$\underbrace{\hat{N}_\gamma :\ 1 \to 0}_{\text{occupation-gone}} \qquad \Big| \qquad \underbrace{\eta_{\rm abs} \to 0}_{\text{identity-gone}}$$

rigid, universal, observer-independent | soft, variable, access-relative

The drama lives in the one quantity that is neither rigidly conserved-and-localized, like energy and fermion number, nor freely annihilable, like photon number, but globally conserved and freely localized: identity.

13 Connections

The structure is not peculiar to absorption. A globally conserved total, a locally non-recoverable part, and an inaccessible reservoir D is the skeleton of the black-hole information problem, with $\eta_{\rm abs}$ measuring how much infalling identity stays locally accessible [8, 9]. The leakage $\mathcal{L}_{\rm abs}$ is decoherence in the einselection sense [11]: the environment carries off the records the atom fails to keep. And because every retained bit is a bit D does not get, $\eta_{\rm abs}$ doubles as a which-path knob: the absorption analog of visibility–distinguishability complementarity [10]. The mathematics is familiar; the asymmetry is the point.

14 Swept-overlap prediction

The framework need not stay a lens. Once a real transition is named, the toy-model overlap $c_A = |\langle e_0 \,|\, e_1 \rangle|$ of §8 is fixed by selection rules. The genuine falsifiable claim is not just the two endpoints, but the whole swept curve

$$\eta_{\rm abs} = h_2\left(\frac{1 + c_A}{2}\right),$$

where an unmodeled leakage channel would show up as a deviation from the functional form.

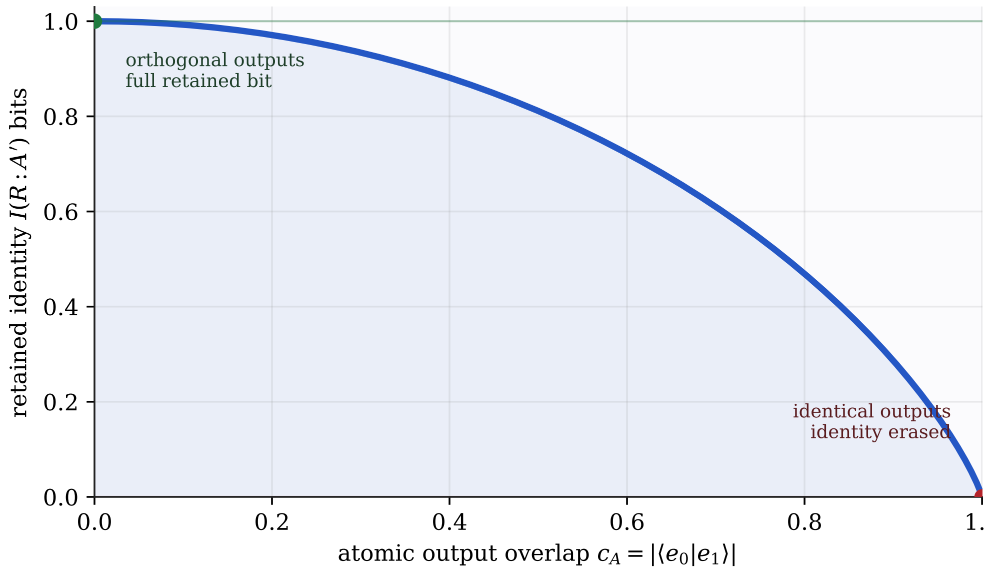

Figure 1: Two-state identity retention. For a balanced binary label, $I(R{:}A') = h_2((1 + c_A)/2)$ with $c_A = |\langle e_0 \,|\, e_1\rangle|$. Orthogonal outputs retain one bit; identical outputs retain none.

Let $\{|e_m\rangle\}$ be the internal excited manifold of dimension d_e. The familiar endpoint cases are consistency checks. A polarization qubit absorbed on a $J_g{=}0 \to J_e{=}1$ transition sends $\sigma^\pm$ to orthogonal sublevels $m{=}\pm 1$, so $c_A = 0$ and the atom should carry the qubit [12]. Time-bin or within-linewidth spectral alternatives on the same σ^+ line both drive $|e_{+1}\rangle$, so $c_A = 1$ and the internal state should retain no which-pulse information. Those endpoints confirm expected behavior. The high-risk prediction is the intermediate regime: a controlled sweep of c_A should trace Fig. 1.

15 Internal-dimension no-go

The same internal carrier gives an alphabet-size no-go. In the classical-label setting of §7, define

$$\eta_{\text{int}} := \frac{I(R{:}A'_{\text{int}})}{\mathcal{J}_{\text{abs}}}, \qquad \mathcal{J}_{\text{abs}} := I(R{:}A\gamma).$$

Proposition 2 (Internal-dimension bound)**.** *If the retained record is read only from an internal excited manifold of dimension d_e, then*

$$\eta_{\text{int}} \le \frac{\log_2 d_e}{\mathcal{J}_{\text{abs}}}.$$

Indeed, in the classical/Holevo setting, $I(R{:}A'_{\text{int}}) = \chi_{\text{int}}$ for the ensemble $\{p_x, \rho^x_{A'_{\text{int}}}\}$, and

$$I(R{:}A'_{\text{int}}) = \chi_{\text{int}} \le S\left(\sum_x p_x \rho^x_{A'_{\text{int}}}\right) \le \log_2 d_e,$$

division by $\mathcal{J}_{\text{abs}} > 0$ gives the bound. Thus $\eta_{\text{int}} = 1$ is reachable only if $\mathcal{J}_{\text{abs}} \le \log_2 d_e$: the identity must fit in the atom's internal manifold. If $\mathcal{J}_{\text{abs}} > \log_2 d_e$, then $\eta_{\text{int}} < 1$ and is bounded by $\log_2 d_e/\mathcal{J}_{\text{abs}}$. For fixed d_e the upper envelope tends to zero as $\mathcal{J}_{\text{abs}}$ grows. A finite-dimensional internal atom cannot localize continuous photonic identity, such as arrival time, spectral envelope, or transverse mode, in A'_{int} alone; the excess is forced into D by dimension counting, independent of coupling strength once this carrier is fixed.

For N perfectly distinguishable, balanced photon alternatives, the initial classical-reference state has $\mathcal{J}_{\text{abs}} = H(X) = \log_2 N$: the photon states are orthogonal, so the reference label is fully recoverable before absorption. With $d_e = 2$, one internal bit is available and

$$\eta_{\text{int}} \leq \frac{1}{\log_2 N}.$$

N	$\mathcal{J}_{\text{abs}} = \log_2 N$	η_{int} envelope for $d_e = 2$
2	1 bit	≤ 1.00
4	2 bits	≤ 0.50
16	4 bits	≤ 0.25

The experimental use is as a saturation benchmark. On one transition, increase N, reconstruct the internal ensemble, and compute χ_{int}. An ideal internal memory saturates $\chi_{\text{int}} = \log_2 d_e$ once the alphabet exceeds the manifold capacity, so $\eta_{\text{int}} = \chi_{\text{int}}/\log_2 N$ approaches the $\log_2 d_e/\log_2 N$ envelope. A shortfall below that envelope quantifies leakage to D in internal-carrier units; a value above the ceiling indicates tomography error or a misidentified carrier dimension, not a physical violation of the bound. The same single-atom and single-ion tomography infrastructure applies if runs are performed at two or more alphabet sizes [13, 14, 15].

The endpoint and ceiling claims can be checked retrospectively, and the envelope uses the same workflow once two or more alphabet sizes are run. The closest physical match is photon-to-atom state transfer, where an absorbed photon's polarization is mapped onto the spin state of a single $^{40}Ca^{+}$ ion [13]. Related single-atom and single-ion experiments publish tomography-level or figure-source data suitable for reanalysis [14, 15]. For a classical input label x, reconstruct $\rho^x_{A'_{\text{int}}}$ and compute

$$\chi_{\text{int}} = S\left(\sum_x p_x \rho^x_{A'_{\text{int}}}\right) - \sum_x p_x S\left(\rho^x_{A'_{\text{int}}}\right).$$

For a balanced binary test,

$$\chi_{\text{int}} = S\left(\frac{\rho^0_{A'_{\text{int}}} + \rho^1_{A'_{\text{int}}}}{2}\right) - \frac{1}{2} S\left(\rho^0_{A'_{\text{int}}}\right) - \frac{1}{2} S\left(\rho^1_{A'_{\text{int}}}\right).$$

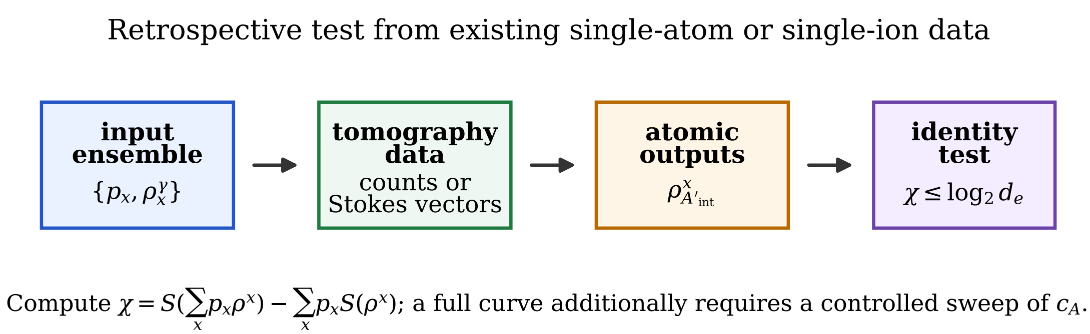

Figure 2: Existing-data consistency checks. Published tomography gives $\{p_x, \rho^x_{A'_{\text{int}}}\}$ and hence χ_{int}. Existing data can check endpoint retention and $\chi_{\text{int}} \leq \log_2 d_e$; multi-alphabet runs measure saturation or shortfall relative to the $\log_2 d_e/\log_2 N$ envelope, and the prediction in Fig. 1 requires a controlled sweep of c_A.

Internal-state tomography should recover a polarization qubit when alternatives populate orthogonal internal states, recover no which-pulse identity when time-bin alternatives drive the same internal state, and never exceed $\log_2 d_e$. These are consistency checks. The curve is sharper: which-pulse retention at fixed frequency and polarization, or a swept-overlap deviation from Fig. 1, would falsify the identification of the localization fraction with internal-state distinguishability. The ceiling and its alphabet envelope are bounds; their role is to make high-alphabet internal localization impossible without extra carrier degrees of freedom.

Conclusion. The framing result is the localization asymmetry. Energy and identity obey parallel global accounting across $A'D$, but energy remains recoil-pinned near unity while identity can localize, leak, or vanish from the atom depending on the absorption channel. The concrete result is the internal-dimension bound: $\eta_{\rm int} \le \log_2 d_e/\mathcal{J}_{\rm abs}$, giving the $\log_2 d_e/\log_2 N$ envelope for balanced N-letter photons. The decomposition $\mathcal{J}_{\rm abs} = \mathcal{I}_{\rm ret} + \mathcal{L}_{\rm abs}$ makes the asymmetry explicit, $\eta_{\rm abs}$ measures it, and the swept c_A curve is where the framing can fail.

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